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Sec 2

$$\overline{GH}(z) = \frac{K(z-0.2)}{(z-1)(z+0.6)^2}$$

Find K for Stability

$\boxed{1}$ Bilinear Transformation

Ch.equation $\quad 1+\overline{GH}(z)=0$

$$(z-1)(z+0.6)^2+K(z-0.2)=0$$

$$(z-1)(z^2+1.2z+0.36)+K(z-0.2)=0$$

$$z^3+0.2z^2+(K-0.84)z+(-0.2K-0.36)=0$$

$$z=\frac{1+r}{1-r}$$

$$\left(\frac{1+r}{1-r}\right)^3+0.2\left(\frac{1+r}{1-r}\right)^2+(K-0.84)\left(\frac{1+r}{1-r}\right)+(-0.2K-0.36)=0$$

$$(1+r)^3+0.2(1+r)^2(1-r)+(K-0.84)(1+r)(1-r)^2$$
$$+(-0.2K-0.36)(1-r)^3=0$$

$$(1+r)^3=r^3+3r^2+3r+1$$
$$(1-r)^3=-r^3+3r^2-3r+1$$
$$(1-r)(1+r)^2=-r^3-r^2+r+1$$
$$(1+r)(1-r)^2=r^3-r^2-r+1$$

$$r^3+3r^2+3r+1-0.2r^3-0.2r^2+0.2r+0.2$$
$$+Kr^3-Kr^2-Kr-K-0.84r^3+0.84r^2$$
$$+0.84r-0.84+0.2Kr^3-0.6Kr^2$$
$$+0.6Kr-0.2K+0.36r^3-1.08r^2$$
$$+1.08r-0.36=0$$

$$(0.32+1.2)r^3+(2.56-1.6K)r^2$$
$$+(5.12-0.4K)r+0.8K=0$$

Routh Array

r^3	$(0.32+1.2K)$	$(5.12-0.4K)$
r^2	$(2.56-1.6K)$	$0.8K$
r^1	$\frac{(2.56-1.6K)(5.12-0.4K)-0.8K(0.32+1.2K)}{(2.56-1.6K)}$	
r^0	$0.8K$	

For stability all first elements must have the same sign positive or negative

* all positive

$0.8K>0 \quad \rightarrow \quad K>0$

and

$0.32+1.2K>0 \quad \rightarrow \quad K>-0.26$

and

$2.56-1.6K>0 \quad \rightarrow \quad K<1.6$

and

$$\frac{(2.56-1.6K)(3.12-0.4K)-0.8K(0.32+1.2K)}{2.56-1.6K}>0$$

$$-(0.32K^2+9.472K-13.1072)>0$$

$$(K-1.32)(K+30.92)<0$$

$K-1.32>0$

$K>1.32$

and

$K+30.92<0$

$K<-30.92$

$K\in\phi$

OR

$K-1.32<0$

$K<1.32$

and

$K+30.92>0$

$K>-30.92$

$-30.92<K<1.32$

$$0.30.92<K<1.32$$

K for Stability

$$0<K<1.32$$

$\boxed{2}$ Jury method

Ch.equation

$$F(z)=z^3+0.2z^2+(K-0.84)z$$
$$+(-0.2K-0.36).$$

① $F(1)>0$

$$1+0.2+K-0.84-0.2K-0.36>0$$

$$0.8K>0$$

$$\boxed{K>0}$$

② $F(-1)(-1)^n>0$

$$1.2K+0.32>0$$

$$\boxed{K>-0.267}$$

③ $|a_0| < |a_n|$

$$|+0.2K + 0.36| < 1$$

$$-1 < +0.2K + 0.36 < 1$$

$$\boxed{-6.8 < K < 3.2}$$

	z^0	z^1	z^2	z^3
1	$(-0.2K-0.36)$	$(K-0.84)$	0.2	1
2	1	0.2	$(K-0.84)$	$(-0.2K-0.36)$
3	b_0	b_1	b_2	

$$b_0 = \begin{vmatrix} -0.2K-0.36 & 1 \ \cancel{K-0.84} \\ 1 & -0.2K-0.36 \end{vmatrix}$$

$$b_0 = (0.2K+0.36)^2 - 1$$

$$b_0 = 0.4K^2 + 0.144K - 0.8704$$

$$b_3 = \begin{vmatrix} -0.2K-0.36 & K-0.84 \\ 1 & 0.2 \end{vmatrix}$$

$$b_3 = (-0.2K-0.36)\,0.2 - (K-0.84)$$

$$b_3 = -1.04K + 0.768$$

④ $|b_0| > |b_{n-1}|$

$$|0.4K^2 + 0.144K - 0.8704| > |-1.04+0.7$$

$$\left|\frac{0.4K^2+0.144K-0.8704}{-1.04K+0.768}\right| > 1$$

$$|M| > 1$$

$$M > 1 \text{ or } M < -1$$

ⓐ $M > 1$

$$\frac{0.4K^2+0.144K-0.8704}{-1.04K+0.768} - 1 > 0$$

$$\frac{0.4K^2+0.144K-0.8704+1.04K-0.768}{-1.04K+0.768} > 0$$

$$\frac{(K-1.027)(K+3.98)}{-1.04K+0.768} > 0$$

(i) $K > 1.027$ and $K > -3.98$ and $K < 0.738 \rightarrow \phi$

(ii) $K > 1.027$ and $K < -3.98$ and $K > .738 \rightarrow \phi$

(iii) $K < 1.027$ and $K > -3.98$ and $K > 0.738 \rightarrow 0.738 < K < 1.027$

(iv) $K < 1.027$ and $K < -3.98$ and $K < 0.738 \rightarrow K < -3.98$

$$0.738 < K < 1.027$$

ⓑ $M < -1$

$$\frac{0.4K^2+0.144K-0.8704}{-1.04K+0.768} + 1 < 0$$

$$\frac{(K-2.11)(K-0.12)}{1.04K-0.768} > 0$$

(i) $K > 2.11$ and $K > 0.12$ and $K > 0.738 \rightarrow K > 2.11$

(ii) $K < 2.11$ and $K < 0.12$ and $K > 0.738 \rightarrow \phi$

(iii) $K < 2.11$ and $K > 0.12$ and $K < 0.738 \rightarrow 0.12 < K < 0.738$

(iv) $K > 2.11$ and $K < 0.12$ and $K < 0.738 \rightarrow \phi$

$0.12 < K < 0.738$ or $K > 2.11$

④ $0.12 < K < 1.027$ or $K > 2.11$

for Stability

$0.12 < K < 1.027$ or ~~and~~ $2.11 < K < 3.2$